

```
<< mathStatica.m
```

<ul style="list-style-type: none"> ■ mathStatica: v1.0 ■ No. of licenses: 2 	<ul style="list-style-type: none"> ■ Registered to: Prof David Rogosa ■ User class: Gold/Professional
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```
(* Nonparametric Delta Method for Variance of Correlation Coefficient
   Efron & Tibshirani section 21.9 result on page 315 *)
```

```
(* for bivariate data (X,Y), n observations, form the vector
   Q = {X, Y, X^2, XY, Y^2} and for q[[i]] = mean(Q[[i]])
   the sample correlation coefficient is given by rfunc below *)
```

```
rfunc[q1_, q2_, q3_, q4_, q5_] := (q4 - q1*q2) /
                                   Sqrt[(q3 - q1^2) * (q5 - q2^2)]
```

```
(* Step 1: Obtain Vr in E&F equation 21.67 with the del function below *)
```

```
ident = IdentityMatrix[5];
```

```
delCorr[q1_, q2_, q3_, q4_, q5_] :=
  Table[FullSimplify[Derivative[ident[[i, 1]], ident[[i, 2]], ident[[i, 3]],
                    ident[[i, 4]], ident[[i, 5]]][rfunc][q1, q2, q3, q4, q5]], {i, 1, 5}]
```

```
delCorr[q1, q2, q3, q4, q5]
```

$$\left\{ \frac{(q_2 q_3 - q_1 q_4) (q_2^2 - q_5)}{((q_1^2 - q_3) (q_2^2 - q_5))^{3/2}}, \frac{(q_1^2 - q_3) (-q_2 q_4 + q_1 q_5)}{((q_1^2 - q_3) (q_2^2 - q_5))^{3/2}}, \right. \\ \left. - \frac{(-q_1 q_2 + q_4) (-q_2^2 + q_5)}{2 ((q_1^2 - q_3) (q_2^2 - q_5))^{3/2}}, \frac{1}{\sqrt{(q_1^2 - q_3) (q_2^2 - q_5)}}, - \frac{(-q_1^2 + q_3) (-q_1 q_2 + q_4)}{2 ((q_1^2 - q_3) (q_2^2 - q_5))^{3/2}} \right\}$$

```
(* that's a useful bit of calculus done for us instantaneously;
   because the final result, eq 21.70, is expressed in central moments,
   it's useful to reformat the result above in order to convert from
   raw moments to central moments *)
```

```
delCorr[μ1,0, μ0,1, μ2,0, μ1,1, μ0,2]
```

$$\left\{ \frac{(\mu_{0,1}^2 - \mu_{0,2}) (-\mu_{1,0} \mu_{1,1} + \mu_{0,1} \mu_{2,0})}{((\mu_{0,1}^2 - \mu_{0,2}) (\mu_{1,0}^2 - \mu_{2,0}))^{3/2}}, \right. \\ \frac{(\mu_{0,2} \mu_{1,0} - \mu_{0,1} \mu_{1,1}) (\mu_{1,0}^2 - \mu_{2,0})}{((\mu_{0,1}^2 - \mu_{0,2}) (\mu_{1,0}^2 - \mu_{2,0}))^{3/2}}, - \frac{(-\mu_{0,1} + \mu_{0,2}) (-\mu_{0,1} \mu_{1,0} + \mu_{1,1})}{2 ((\mu_{0,1}^2 - \mu_{0,2}) (\mu_{1,0}^2 - \mu_{2,0}))^{3/2}}, \\ \left. \frac{1}{\sqrt{(\mu_{0,1}^2 - \mu_{0,2}) (\mu_{1,0}^2 - \mu_{2,0})}}, - \frac{(-\mu_{0,1} \mu_{1,0} + \mu_{1,1}) (-\mu_{1,0}^2 + \mu_{2,0})}{2 ((\mu_{0,1}^2 - \mu_{0,2}) (\mu_{1,0}^2 - \mu_{2,0}))^{3/2}} \right\}$$

```
(* transformation from raw to central moments is a two-step
   procedure, using the facilities in mathStatica *)
```

```
?RawToCumulant
```

```
RawToCumulant[r] expresses the rth raw moment μr in terms of
  cumulants κi. To obtain a multivariate conversion, let r be a list of integers.
```

```
?CumulantToCentral
```

```
CumulantToCentral[r] expresses the rth cumulant κr in terms of central
  moments μi. To obtain a multivariate conversion, let r be a list of integers.
```

```
rulesDelRawtoK = {RawToCumulant[{0, 1}], RawToCumulant[{0, 2}],
  RawToCumulant[{1, 0}], RawToCumulant[{1, 1}], RawToCumulant[{2, 0}]}
```

```
{μ0,1 → κ0,1, μ0,2 → κ0,12 + κ0,2, μ1,0 → κ1,0, μ1,1 → κ0,1 κ1,0 + κ1,1, μ2,0 → κ1,02 + κ2,0}
```

```
delCorrinK = delCorr[ $\mu_{1,0}$ ,  $\mu_{0,1}$ ,  $\mu_{2,0}$ ,  $\mu_{1,1}$ ,  $\mu_{0,2}$ ] /. rulesDelRawtoK
```

$$\left\{ -\frac{\kappa_{0,2} (-\kappa_{1,0} (\kappa_{0,1} \kappa_{1,0} + \kappa_{1,1}) + \kappa_{0,1} (\kappa_{1,0}^2 + \kappa_{2,0}))}{(\kappa_{0,2} \kappa_{2,0})^{3/2}}, -\frac{((\kappa_{0,1}^2 + \kappa_{0,2}) \kappa_{1,0} - \kappa_{0,1} (\kappa_{0,1} \kappa_{1,0} + \kappa_{1,1})) \kappa_{2,0}}{(\kappa_{0,2} \kappa_{2,0})^{3/2}}, \right. \\ \left. -\frac{\kappa_{0,2} \kappa_{1,1}}{2 (\kappa_{0,2} \kappa_{2,0})^{3/2}}, \frac{1}{\sqrt{\kappa_{0,2} \kappa_{2,0}}}, -\frac{\kappa_{1,1} \kappa_{2,0}}{2 (\kappa_{0,2} \kappa_{2,0})^{3/2}} \right\}$$

```
FullSimplify[%]
```

$$\left\{ \frac{\kappa_{0,2} (\kappa_{1,0} \kappa_{1,1} - \kappa_{0,1} \kappa_{2,0})}{(\kappa_{0,2} \kappa_{2,0})^{3/2}}, \frac{(-\kappa_{0,2} \kappa_{1,0} + \kappa_{0,1} \kappa_{1,1}) \kappa_{2,0}}{(\kappa_{0,2} \kappa_{2,0})^{3/2}}, \right. \\ \left. -\frac{\kappa_{0,2} \kappa_{1,1}}{2 (\kappa_{0,2} \kappa_{2,0})^{3/2}}, \frac{1}{\sqrt{\kappa_{0,2} \kappa_{2,0}}}, -\frac{\kappa_{1,1} \kappa_{2,0}}{2 (\kappa_{0,2} \kappa_{2,0})^{3/2}} \right\}$$

```
rulesDelKtoC = {CumulantToRaw[{1, 0}], CumulantToRaw[{0, 1}],  
CumulantToCentral[{2, 0}], CumulantToCentral[{1, 1}], CumulantToCentral[{0, 2}]}
```

```
{ $\kappa_{1,0} \rightarrow \mu_{1,0}$ ,  $\kappa_{0,1} \rightarrow \mu_{0,1}$ ,  $\kappa_{2,0} \rightarrow \mu_{2,0}$ ,  $\kappa_{1,1} \rightarrow \mu_{1,1}$ ,  $\kappa_{0,2} \rightarrow \mu_{0,2}$ }
```

```
delCorrinC = FullSimplify[delCorrinK /. rulesDelKtoC]
```

$$\left\{ \frac{\mu_{0,2} (-\mu_{2,0} \mu_{0,1} + \mu_{1,1} \mu_{1,0})}{(\mu_{0,2} \mu_{2,0})^{3/2}}, \frac{\mu_{2,0} (\mu_{1,1} \mu_{0,1} - \mu_{0,2} \mu_{1,0})}{(\mu_{0,2} \mu_{2,0})^{3/2}}, \right. \\ \left. -\frac{\mu_{0,2} \mu_{1,1}}{2 (\mu_{0,2} \mu_{2,0})^{3/2}}, \frac{1}{\sqrt{\mu_{0,2} \mu_{2,0}}}, -\frac{\mu_{1,1} \mu_{2,0}}{2 (\mu_{0,2} \mu_{2,0})^{3/2}} \right\}$$

```
(* Step 2: Build up the 5x5 covariance matrix of the Q vector  
I do so element by element for simple exposition; there are  
more elegant forms for this *)
```

```
sig11 =  $\mu_{2,0}$ ; sig22 =  $\mu_{0,2}$ ; sig12 = sig21 =  $\mu_{1,1}$ ;  
sig33 =  $\sqrt{(-\mu_{1,0} \mu_{2,0} + \mu_{3,0})^2 + \mu_{1,1} \mu_{2,0}}$ ;  
sig55 =  $\sqrt{(-\mu_{0,1} \mu_{2,0} + \mu_{2,1})^2 + \mu_{0,1} \mu_{2,0}}$ ;  
sig44 =  $\sqrt{(-\mu_{1,1} \mu_{2,0} + \mu_{3,1})^2 + \mu_{1,1} \mu_{2,0}}$ ;  
sig13 = sig31 =  $-\mu_{1,0} \mu_{2,0} + \mu_{3,0}$ ; sig25 = sig52 =  $-\mu_{0,1} \mu_{2,0} + \mu_{0,3}$ ;  
sig14 = sig41 =  $-\mu_{1,1} \mu_{1,0} + \mu_{2,1}$ ;  
sig15 = sig51 =  $-\mu_{0,2} \mu_{1,0} + \mu_{1,2}$ ; sig23 = sig32 =  $-\mu_{2,0} \mu_{0,1} + \mu_{2,1}$ ;  
sig24 = sig42 =  $-\mu_{1,1} \mu_{0,1} + \mu_{1,2}$ ; sig34 = sig43 =  $-\mu_{1,1} \mu_{2,0} + \mu_{3,1}$ ;  
sig35 = sig53 =  $-\mu_{0,2} \mu_{2,0} + \mu_{2,2}$ ; sig45 = sig54 =  $-\mu_{1,1} \mu_{0,2} + \mu_{1,3}$ ;
```

```
sigRaw = {{sig11, sig12, sig13, sig14, sig15},  
{sig21, sig22, sig23, sig24, sig25}, {sig31, sig32, sig33, sig34, sig35},  
{sig41, sig42, sig43, sig44, sig45}, {sig51, sig52, sig53, sig54, sig55}}
```

$$\begin{pmatrix} \mu_{2,0} & \mu_{1,1} & -\mu_{1,0} \mu_{2,0} + \mu_{3,0} & -\mu_{1,0} \mu_{1,1} + \mu_{2,1} & -\mu_{0,2} \mu_{1,0} + \mu_{1,2} \\ \mu_{1,1} & \mu_{0,2} & -\mu_{0,1} \mu_{2,0} + \mu_{2,1} & -\mu_{0,1} \mu_{1,1} + \mu_{1,2} & -\mu_{0,1} \mu_{0,2} + \mu_{0,3} \\ -\mu_{1,0} \mu_{2,0} + \mu_{3,0} & -\mu_{0,1} \mu_{2,0} + \mu_{2,1} & -\mu_{2,0}^2 + \mu_{4,0} & -\mu_{1,1} \mu_{2,0} + \mu_{3,1} & -\mu_{0,2} \mu_{2,0} + \mu_{2,2} \\ -\mu_{1,0} \mu_{1,1} + \mu_{2,1} & -\mu_{0,1} \mu_{1,1} + \mu_{1,2} & -\mu_{1,1} \mu_{2,0} + \mu_{3,1} & -\mu_{1,1}^2 + \mu_{2,2} & -\mu_{0,2} \mu_{1,1} + \mu_{1,3} \\ -\mu_{0,2} \mu_{1,0} + \mu_{1,2} & -\mu_{0,1} \mu_{0,2} + \mu_{0,3} & -\mu_{0,2} \mu_{2,0} + \mu_{2,2} & -\mu_{0,2} \mu_{1,1} + \mu_{1,3} & -\mu_{0,2}^2 + \mu_{0,4} \end{pmatrix}$$

```
FullSimplify[%]
```

$$\begin{pmatrix} \mu_{2,0} & \mu_{1,1} & -\mu_{1,0} \mu_{2,0} + \mu_{3,0} & -\mu_{1,0} \mu_{1,1} + \mu_{2,1} & -\mu_{0,2} \mu_{1,0} + \mu_{1,2} \\ \mu_{1,1} & \mu_{0,2} & -\mu_{0,1} \mu_{2,0} + \mu_{2,1} & -\mu_{0,1} \mu_{1,1} + \mu_{1,2} & -\mu_{0,1} \mu_{0,2} + \mu_{0,3} \\ -\mu_{1,0} \mu_{2,0} + \mu_{3,0} & -\mu_{0,1} \mu_{2,0} + \mu_{2,1} & -\mu_{2,0}^2 + \mu_{4,0} & -\mu_{1,1} \mu_{2,0} + \mu_{3,1} & -\mu_{0,2} \mu_{2,0} + \mu_{2,2} \\ -\mu_{1,0} \mu_{1,1} + \mu_{2,1} & -\mu_{0,1} \mu_{1,1} + \mu_{1,2} & -\mu_{1,1} \mu_{2,0} + \mu_{3,1} & -\mu_{1,1}^2 + \mu_{2,2} & -\mu_{0,2} \mu_{1,1} + \mu_{1,3} \\ -\mu_{0,2} \mu_{1,0} + \mu_{1,2} & -\mu_{0,1} \mu_{0,2} + \mu_{0,3} & -\mu_{0,2} \mu_{2,0} + \mu_{2,2} & -\mu_{0,2} \mu_{1,1} + \mu_{1,3} & -\mu_{0,2}^2 + \mu_{0,4} \end{pmatrix}$$

```
rulesSigmaRawtoK = Flatten[Table[RawToCumulant[{i, j}], {i, 0, 4}, {j, 0, 4}]];
```

```
sigK = FullSimplify[sigRaw /. rulesSigmaRawtoK];
```

```

rulesSigmaKtoC = {CumulantToRaw[{1, 0}], CumulantToRaw[{0, 1}],
  CumulantToCentral[{2, 0}], CumulantToCentral[{1, 1}],
  CumulantToCentral[{0, 2}], CumulantToCentral[{3, 0}],
  CumulantToCentral[{0, 3}], CumulantToCentral[{2, 1}],
  CumulantToCentral[{1, 2}], CumulantToCentral[{2, 2}], CumulantToCentral[{3, 1}],
  CumulantToCentral[{4, 0}], CumulantToCentral[{0, 4}], CumulantToCentral[{1, 3}],
  CumulantToCentral[{2, 3}], CumulantToCentral[{4, 1}],
  CumulantToCentral[{3, 2}], CumulantToCentral[{1, 4]}}

```

$$\left\{ \begin{aligned} \kappa_{1,0} &\rightarrow \acute{\mu}_{1,0}, \kappa_{0,1} \rightarrow \acute{\mu}_{0,1}, \kappa_{2,0} \rightarrow \mu_{2,0}, \kappa_{1,1} \rightarrow \mu_{1,1}, \kappa_{0,2} \rightarrow \mu_{0,2}, \kappa_{3,0} \rightarrow \mu_{3,0}, \kappa_{0,3} \rightarrow \mu_{0,3}, \\ \kappa_{2,1} &\rightarrow \mu_{2,1}, \kappa_{1,2} \rightarrow \mu_{1,2}, \kappa_{2,2} \rightarrow -2 \mu_{1,1}^2 - \mu_{0,2} \mu_{2,0} + \mu_{2,2}, \kappa_{3,1} \rightarrow -3 \mu_{1,1} \mu_{2,0} + \mu_{3,1}, \\ \kappa_{4,0} &\rightarrow -3 \mu_{2,0}^2 + \mu_{4,0}, \kappa_{0,4} \rightarrow -3 \mu_{0,2}^2 + \mu_{0,4}, \kappa_{1,3} \rightarrow -3 \mu_{0,2} \mu_{1,1} + \mu_{1,3}, \\ \kappa_{2,3} &\rightarrow -6 \mu_{1,1} \mu_{1,2} - \mu_{0,3} \mu_{2,0} - 3 \mu_{0,2} \mu_{2,1} + \mu_{2,3}, \kappa_{4,1} \rightarrow -6 \mu_{2,0} \mu_{2,1} - 4 \mu_{1,1} \mu_{3,0} + \mu_{4,1}, \\ \kappa_{3,2} &\rightarrow -3 \mu_{1,2} \mu_{2,0} - 6 \mu_{1,1} \mu_{2,1} - \mu_{0,2} \mu_{3,0} + \mu_{3,2}, \kappa_{1,4} \rightarrow -4 \mu_{0,3} \mu_{1,1} - 6 \mu_{0,2} \mu_{1,2} + \mu_{1,4} \end{aligned} \right\}$$

```
sigCentral = FullSimplify[sigK /. rulesSigmaKtoC];
```

```
TableForm[sigCentral[[1]]]
```

$$\begin{array}{l} \mu_{2,0} \\ \mu_{1,1} \\ \mu_{3,0} + 2 \mu_{2,0} \acute{\mu}_{1,0} \\ \mu_{2,1} + \mu_{2,0} \acute{\mu}_{0,1} + \mu_{1,1} \acute{\mu}_{1,0} \\ \mu_{1,2} + 2 \mu_{1,1} \acute{\mu}_{0,1} \end{array}$$

```
TableForm[sigCentral[[2]]]
```

$$\begin{array}{l} \mu_{1,1} \\ \mu_{0,2} \\ \mu_{2,1} + 2 \mu_{1,1} \acute{\mu}_{1,0} \\ \mu_{1,2} + \mu_{1,1} \acute{\mu}_{0,1} + \mu_{0,2} \acute{\mu}_{1,0} \\ \mu_{0,3} + 2 \mu_{0,2} \acute{\mu}_{0,1} \end{array}$$

```
TableForm[sigCentral[[3]]]
```

$$\begin{array}{l} \mu_{3,0} + 2 \mu_{2,0} \acute{\mu}_{1,0} \\ \mu_{2,1} + 2 \mu_{1,1} \acute{\mu}_{1,0} \\ -\mu_{2,0}^2 + \mu_{4,0} + 4 \mu_{3,0} \acute{\mu}_{1,0} + 4 \mu_{2,0} \acute{\mu}_{1,0}^2 \\ \mu_{3,1} + 3 \mu_{2,1} \acute{\mu}_{1,0} + \acute{\mu}_{0,1} (\mu_{3,0} + 2 \mu_{2,0} \acute{\mu}_{1,0}) - \mu_{1,1} (\mu_{2,0} - 2 \acute{\mu}_{1,0}^2) \\ -\mu_{0,2} \mu_{2,0} + \mu_{2,2} + 2 \mu_{2,1} \acute{\mu}_{0,1} + 2 (\mu_{1,2} + 2 \mu_{1,1} \acute{\mu}_{0,1}) \acute{\mu}_{1,0} \end{array}$$

```
TableForm[sigCentral[[4]]]
```

$$\begin{array}{l} \mu_{2,1} + \mu_{2,0} \acute{\mu}_{0,1} + \mu_{1,1} \acute{\mu}_{1,0} \\ \mu_{1,2} + \mu_{1,1} \acute{\mu}_{0,1} + \mu_{0,2} \acute{\mu}_{1,0} \\ \mu_{3,1} + 3 \mu_{2,1} \acute{\mu}_{1,0} + \acute{\mu}_{0,1} (\mu_{3,0} + 2 \mu_{2,0} \acute{\mu}_{1,0}) - \mu_{1,1} (\mu_{2,0} - 2 \acute{\mu}_{1,0}^2) \\ -\mu_{1,1}^2 + \mu_{2,2} + 2 \mu_{2,1} \acute{\mu}_{0,1} + \mu_{2,0} \acute{\mu}_{0,1}^2 + 2 (\mu_{1,2} + \mu_{1,1} \acute{\mu}_{0,1}) \acute{\mu}_{1,0} + \mu_{0,2} \acute{\mu}_{1,0}^2 \\ -\mu_{0,2} \mu_{1,1} + \mu_{1,3} + 3 \mu_{1,2} \acute{\mu}_{0,1} + 2 \mu_{1,1} \acute{\mu}_{0,1}^2 + (\mu_{0,3} + 2 \mu_{0,2} \acute{\mu}_{0,1}) \acute{\mu}_{1,0} \end{array}$$

```
TableForm[sigCentral[[5]]]
```

$$\begin{array}{l} \mu_{1,2} + 2 \mu_{1,1} \acute{\mu}_{0,1} \\ \mu_{0,3} + 2 \mu_{0,2} \acute{\mu}_{0,1} \\ -\mu_{0,2} \mu_{2,0} + \mu_{2,2} + 2 \mu_{2,1} \acute{\mu}_{0,1} + 2 (\mu_{1,2} + 2 \mu_{1,1} \acute{\mu}_{0,1}) \acute{\mu}_{1,0} \\ -\mu_{0,2} \mu_{1,1} + \mu_{1,3} + 3 \mu_{1,2} \acute{\mu}_{0,1} + 2 \mu_{1,1} \acute{\mu}_{0,1}^2 + (\mu_{0,3} + 2 \mu_{0,2} \acute{\mu}_{0,1}) \acute{\mu}_{1,0} \\ -\mu_{0,2}^2 + \mu_{0,4} + 4 \mu_{0,3} \acute{\mu}_{0,1} + 4 \mu_{0,2} \acute{\mu}_{0,1}^2 \end{array}$$

```
Dimensions[sigCentral]
```

```
{5, 5}
```

```
(* Step 3: Now form vector-matrix product  $\nabla' \Sigma \nabla$  E&F eq 21.67 *)
```

```
result = delCorrinC.sigCentral.delCorrinC // FullSimplify
```

$$\frac{1}{4 \mu_{0,2}^3 \mu_{2,0}^3} (\mu_{0,4} \mu_{1,1}^2 \mu_{2,0}^2 + \mu_{0,2} (4 \mu_{0,2} \mu_{2,0}^2 \mu_{2,2} - 4 \mu_{1,1} \mu_{2,0} (\mu_{1,3} \mu_{2,0} + \mu_{0,2} \mu_{3,1}) + \mu_{1,1}^2 (2 \mu_{2,0} \mu_{2,2} + \mu_{0,2} \mu_{4,0})))$$

```
Expand[result]
```

$$\frac{\mu_{0,4} \mu_{1,1}^2}{4 \mu_{0,2}^3 \mu_{2,0}^2} - \frac{\mu_{1,1} \mu_{1,3}}{\mu_{0,2}^2 \mu_{2,0}^2} + \frac{\mu_{1,1}^2 \mu_{2,2}}{2 \mu_{0,2}^2 \mu_{2,0}^2} + \frac{\mu_{2,2}}{\mu_{0,2} \mu_{2,0}} - \frac{\mu_{1,1} \mu_{3,1}}{\mu_{0,2} \mu_{2,0}^2} + \frac{\mu_{1,1}^2 \mu_{4,0}}{4 \mu_{0,2} \mu_{2,0}^3}$$

```
(*this is n*result in E&F eq 21.70; to match the algebraic form need to pull out the factor corr^2/4 *)
```

```
Expand[result * 4 / (\mu_{1,1}^2 / (\mu_{0,2} \mu_{2,0}))]
```

$$\frac{\mu_{0,4}}{\mu_{0,2}^2} - \frac{4 \mu_{1,3}}{\mu_{0,2} \mu_{1,1}} + \frac{4 \mu_{2,2}}{\mu_{1,1}^2} + \frac{2 \mu_{2,2}}{\mu_{0,2} \mu_{2,0}} - \frac{4 \mu_{3,1}}{\mu_{1,1} \mu_{2,0}} + \frac{\mu_{4,0}}{\mu_{2,0}^2}$$

```
(* the above gives the terms within the brackets in eq 21.70 *)
```

Textbook pages attached

21.9 The delta method

The delta method is a special technique for variance estimation that is applicable to statistics that are functions of observed averages. Suppose that we can write

$$\hat{\theta}(X_1, X_2, \dots, X_n) = r(\bar{Q}_1, \bar{Q}_2, \dots, \bar{Q}_A), \quad (21.59)$$

where $r(\cdot, \cdot, \dots, \cdot)$ is a known function and

$$\bar{Q}_a = \frac{1}{n} \sum_1^n Q_a(X_i). \quad (21.60)$$

The simplest example is the mean, for which $Q_a(X_i) = X_i$; for the correlation we take

$$r(\bar{Q}_1, \bar{Q}_2, \bar{Q}_3, \bar{Q}_4, \bar{Q}_5) = \frac{\bar{Q}_4 - \bar{Q}_1 \bar{Q}_2}{[\bar{Q}_3 - \bar{Q}_1^2]^{1/2} [\bar{Q}_5 - \bar{Q}_2^2]^{1/2}} \quad (21.61)$$

with $X = (Y, Z)$, $Q_1(X) = Y$, $Q_2(X) = Z$, $Q_3(X) = Y^2$, $Q_4(X) = YZ$, $Q_5(X) = Z^2$.

The idea behind the delta method is the following. Suppose we have a random variable U with mean μ and variance σ^2 , and we seek the variance of a one-to-one function of U , say $g(U)$. By expanding $g(t)$ in a one term Taylor series about $t = \mu$ we have

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