

In[5]:= **s[r\_] := 1 - r^2**

In[15]:= **f[r\_] := (1 / Sqrt[s[r]]) \*  
Exp[(u\*v) \* r / s[r] - ((u^2 + v^2) / 2) \* r^2 / s[r]] - 1**

In[16]:= **f[r]**

Out[16]=  $-1 + \frac{e^{\frac{r u v}{1-r^2} - \frac{r^2 (u^2+v^2)}{2(1-r^2)}}}{\sqrt{1-r^2}}$

In[4]:= **(\* would plot it if I knew values typical for u,v \*)**

In[23]:= **brad5 = Series[f[r], {r, 0, 5}]**

Out[23]=  $u v r + \left( \frac{1}{2} + \frac{1}{2} \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) r^2 +$   
 $\left( \frac{u v}{2} + \frac{1}{3} \left( 3 u v + 2 u v \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} u v \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) r^3 +$   
 $\left( \frac{3}{8} + \frac{1}{4} \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right.$   
 $\left. \frac{1}{4} \left( 3 u^2 v^2 + 4 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right.$   
 $\left. \frac{1}{3} u v \left( 3 u v + 2 u v \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} u v \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) r^4 +$   
 $\left( \frac{3 u v}{8} + \frac{1}{6} \left( 3 u v + 2 u v \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} u v \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) + \right.$   
 $\left. \frac{1}{5} \left( 5 u v + 4 u v \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{3}{2} u v \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right.$   
 $\left. \frac{2}{3} \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( 3 u v + 2 u v \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} u v \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) + \right.$   
 $\left. \frac{1}{4} u v \left( 3 u^2 v^2 + 4 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right.$   
 $\left. \frac{1}{3} u v \left( 3 u v + 2 u v \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} u v \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) r^5 + O[r]^6$

In[18]:= **Series[f[r], {r, 0, 1}]**

Out[18]=  $u v r + O[r]^2$

In[19]:= **Series[f[r], {r, 0, 2}]**

Out[19]=  $u v r + \left( \frac{1}{2} - \frac{u^2}{2} - \frac{v^2}{2} + \frac{u^2 v^2}{2} \right) r^2 + O[r]^3$

In[20]:= **SeriesCoefficient[Series[f[r], {r, 0, 5}], 2]**

Out[20]=  $\frac{1}{2} + \frac{1}{2} \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right)$

In[21]:= **FullSimplify[%]**

Out[21]=  $\frac{1}{2} (-1 + u^2) (-1 + v^2)$

In[22]:= **(\* matches Efron \*)**

In[24]:= **SeriesCoefficient**[brad5, 2]

$$\text{Out}[24]= \frac{1}{2} + \frac{1}{2} \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right)$$

In[25]:= **SeriesCoefficient**[brad5, 3]

$$\text{Out}[25]= \frac{u v}{2} + \frac{1}{3} \left( 3 u v + 2 u v \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} u v \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right)$$

In[29]:= **FullSimplify**[%25]

$$\text{Out}[29]= \frac{1}{6} u (-3 + u^2) v (-3 + v^2)$$

In[30]:= **SeriesCoefficient**[brad5, 4]

$$\begin{aligned} \text{Out}[30]= & \frac{3}{8} + \frac{1}{4} \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \frac{1}{4} \left( 3 u^2 v^2 + 4 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) + \\ & \frac{1}{3} u v \left( 3 u v + 2 u v \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} u v \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \end{aligned}$$

In[31]:= **FullSimplify**[%]

$$\text{Out}[31]= \frac{1}{24} (3 - 6 u^2 + u^4) (3 - 6 v^2 + v^4)$$

In[32]:= **SeriesCoefficient**[brad5, 5]

$$\begin{aligned} \text{Out}[32]= & \frac{3 u v}{8} + \frac{1}{6} \left( 3 u v + 2 u v \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} u v \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) + \\ & \frac{1}{5} \left( 5 u v + 4 u v \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{3}{2} u v \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) + \\ & \frac{2}{3} \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( 3 u v + 2 u v \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} u v \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) + \\ & \frac{1}{4} u v \left( 3 u^2 v^2 + 4 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right. \\ & \left. \frac{1}{3} u v \left( 3 u v + 2 u v \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} u v \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) \end{aligned}$$

In[33]:= **FullSimplify**[%]

$$\text{Out}[33]= \frac{1}{120} u (15 - 10 u^2 + u^4) v (15 - 10 v^2 + v^4)$$

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In[34]:= TableForm[Table[HermiteH[n, r], {n, 5}]]
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Out[34]//TableForm=
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$$\begin{array}{l} 2r \\ -2 + 4r^2 \\ -12r + 8r^3 \\ 12 - 48r^2 + 16r^4 \\ 120r - 160r^3 + 32r^5 \end{array}$$

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In[36]:= (* Sunday part 2 *)
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In[48]:= Plot[Evaluate[{f[r], Normal[brad5]}, {r, 0, .99}] /. {u -> 2, v -> 2}
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In[50]:= Plot[Normal[brad5], {r, 0, .5}] /. {u -> .2, v -> .2}
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In[47]:= f[.5] /. {u -> 1, v -> 1}
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Out[47]= 0.611514
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In[52]:= Normal[brad5] /. {r -> .5, u -> 1, v -> 1}
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Out[52]= 0.603125
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In[67]:= Normal[brad10] /. {r -> .5, u -> 1, v -> 1}
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Out[67]= 0.611391
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In[53]:= brad10 = Series[f[r], {r, 0, 10}]
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$$\begin{aligned} \text{Out[53]} = & uv r + \left( \frac{1}{2} + \frac{1}{2} \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) r^2 + \\ & \left( \frac{uv}{2} + \frac{1}{3} \left( 3uv + 2uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} uv \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) r^3 + \\ & \left( \frac{3}{8} + \frac{1}{4} \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right. \\ & \quad \left. \frac{1}{4} \left( 3u^2 v^2 + 4 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right. \right. \\ & \quad \left. \left. \frac{1}{3} uv \left( 3uv + 2uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} uv \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) \right) r^4 + \\ & \left( \frac{3uv}{8} + \frac{1}{6} \left( 3uv + 2uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} uv \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) + \right. \\ & \quad \left. \frac{1}{5} \left( 5uv + 4uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{3}{2} uv \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right. \right. \\ & \quad \left. \frac{2}{3} \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( 3uv + 2uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} uv \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) + \right. \\ & \quad \left. \frac{1}{4} uv \left( 3u^2 v^2 + 4 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right. \right. \\ & \quad \left. \left. \frac{1}{3} uv \left( 3uv + 2uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} uv \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) \right) \right) r^5 + \\ & \left( \frac{5}{16} + \frac{3}{16} \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \frac{1}{8} \left( 3u^2 v^2 + 4 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( u^2 v^2 + \right. \right. \right. \\ & \quad \left. \left. 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \frac{1}{3} uv \left( 3uv + 2uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} uv \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) \right) + \\ & \quad \left. \frac{1}{6} \left( 5u^2 v^2 + 6 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right. \right. \end{aligned}$$

























$$\begin{aligned}
& \frac{1}{3} u v \left( 3 u v + 2 u v \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} u v \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) + \\
& \frac{1}{6} u v \left( 5 u^2 v^2 + 6 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right. \\
& \quad \left. u v \left( 3 u v + 2 u v \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} u v \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) + \frac{1}{2} \right. \\
& \quad \left. \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( 3 u^2 v^2 + 4 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) + \right. \\
& \quad \left. \frac{1}{3} u v \left( 3 u v + 2 u v \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} u v \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) + \\
& \frac{1}{5} u v \left( 5 u v + 4 u v \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{3}{2} u v \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \frac{2}{3} \right. \\
& \quad \left. \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( 3 u v + 2 u v \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} u v \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) + \\
& \frac{1}{4} u v \left( 3 u^2 v^2 + 4 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right. \\
& \quad \left. \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \frac{1}{3} u v \left( 3 u v + 2 u v \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \right. \right. \\
& \quad \left. \left. \frac{1}{2} u v \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) \right) \right) \right) r^{10} + O[r]^{11}
\end{aligned}$$

In[68]:= **Table[FullSimplify[SeriesCoefficient[brad10, k]], {k, 1, 10}]**

$$\begin{aligned}
\text{Out[68]} = & \left\{ u v, \frac{1}{2} (-1 + u^2) (-1 + v^2), \frac{1}{6} u (-3 + u^2) v (-3 + v^2), \frac{1}{24} (3 - 6 u^2 + u^4) (3 - 6 v^2 + v^4), \right. \\
& \frac{1}{120} u (15 - 10 u^2 + u^4) v (15 - 10 v^2 + v^4), \frac{1}{720} (-15 + 45 u^2 - 15 u^4 + u^6) (-15 + 45 v^2 - 15 v^4 + v^6), \\
& \frac{u (-105 + 105 u^2 - 21 u^4 + u^6) v (-105 + 105 v^2 - 21 v^4 + v^6)}{5040}, \\
& \frac{(105 - 420 u^2 + 210 u^4 - 28 u^6 + u^8) (105 - 420 v^2 + 210 v^4 - 28 v^6 + v^8)}{40320}, \\
& \frac{u (945 - 1260 u^2 + 378 u^4 - 36 u^6 + u^8) v (945 - 1260 v^2 + 378 v^4 - 36 v^6 + v^8)}{362880}, \frac{1}{3628800} \\
& \left. \left( (-945 + 4725 u^2 - 3150 u^4 + 630 u^6 - 45 u^8 + u^{10}) (-945 + 4725 v^2 - 3150 v^4 + 630 v^6 - 45 v^8 + v^{10}) \right) \right\}
\end{aligned}$$

In[69]:= **TableForm[%]**

Out[69]//TableForm=

$$\begin{aligned}
& u v \\
& \frac{1}{2} (-1 + u^2) (-1 + v^2) \\
& \frac{1}{6} u (-3 + u^2) v (-3 + v^2) \\
& \frac{1}{24} (3 - 6 u^2 + u^4) (3 - 6 v^2 + v^4) \\
& \frac{1}{120} u (15 - 10 u^2 + u^4) v (15 - 10 v^2 + v^4) \\
& \frac{1}{720} (-15 + 45 u^2 - 15 u^4 + u^6) (-15 + 45 v^2 - 15 v^4 + v^6) \\
& \frac{u (-105 + 105 u^2 - 21 u^4 + u^6) v (-105 + 105 v^2 - 21 v^4 + v^6)}{5040} \\
& \frac{(105 - 420 u^2 + 210 u^4 - 28 u^6 + u^8) (105 - 420 v^2 + 210 v^4 - 28 v^6 + v^8)}{40320} \\
& \frac{u (945 - 1260 u^2 + 378 u^4 - 36 u^6 + u^8) v (945 - 1260 v^2 + 378 v^4 - 36 v^6 + v^8)}{362880} \\
& \frac{(-945 + 4725 u^2 - 3150 u^4 + 630 u^6 - 45 u^8 + u^{10}) (-945 + 4725 v^2 - 3150 v^4 + 630 v^6 - 45 v^8 + v^{10})}{3628800}
\end{aligned}$$

In[54]:= **SeriesCoefficient**[brad10, 5]

$$\begin{aligned} \text{Out}[54]= & \frac{3uv}{8} + \frac{1}{6} \left( 3uv + 2uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} uv \left( u^2v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) + \\ & \frac{1}{5} \left( 5uv + 4uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{3}{2} uv \left( u^2v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right. \\ & \quad \left. \frac{2}{3} \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( 3uv + 2uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} uv \left( u^2v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) + \\ & \quad \frac{1}{4} uv \left( 3u^2v^2 + 4 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( u^2v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right. \\ & \quad \left. \frac{1}{3} uv \left( 3uv + 2uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} uv \left( u^2v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) \end{aligned}$$

In[55]:= **FullSimplify**[%]

$$\text{Out}[55]= \frac{1}{120} u (15 - 10u^2 + u^4) v (15 - 10v^2 + v^4)$$

In[56]:= (\* same as with brad5 as it should be \*)

In[57]:= **SeriesCoefficient**[brad10, 6]

$$\begin{aligned} \text{Out}[57]= & \frac{5}{16} + \frac{3}{16} \left( u^2v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \\ & \frac{1}{8} \left( 3u^2v^2 + 4 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( u^2v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right. \\ & \quad \left. \frac{1}{3} uv \left( 3uv + 2uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} uv \left( u^2v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) + \\ & \frac{1}{6} \left( 5u^2v^2 + 6 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( u^2v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right. \\ & \quad uv \left( 3uv + 2uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} uv \left( u^2v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) + \\ & \quad \left. \frac{1}{2} \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( 3u^2v^2 + 4 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( u^2v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) + \right. \\ & \quad \left. \frac{1}{3} uv \left( 3uv + 2uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} uv \left( u^2v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) + \\ & \frac{1}{5} uv \left( 5uv + 4uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{3}{2} uv \left( u^2v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right. \\ & \quad \left. \frac{2}{3} \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( 3uv + 2uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} uv \left( u^2v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) + \\ & \quad \frac{1}{4} uv \left( 3u^2v^2 + 4 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( u^2v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right. \\ & \quad \left. \frac{1}{3} uv \left( 3uv + 2uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} uv \left( u^2v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) \end{aligned}$$

In[58]:= **FullSimplify**[%]

$$\text{Out}[58]= \frac{1}{720} (-15 + 45u^2 - 15u^4 + u^6) (-15 + 45v^2 - 15v^4 + v^6)$$



























$$\begin{aligned}
& \frac{4}{3} \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( 3uv + 2uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} uv \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) + \\
& \frac{3}{4} uv \left( 3u^2 v^2 + 4 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right. \\
& \quad \left. \frac{1}{3} uv \left( 3uv + 2uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} uv \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) + \\
& \frac{2}{5} \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( 5uv + 4uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{3}{2} uv \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right. \\
& \quad \left. \frac{2}{3} \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( 3uv + 2uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} uv \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) + \\
& \frac{1}{4} uv \left( 3u^2 v^2 + 4 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right. \\
& \quad \left. \frac{1}{3} uv \left( 3uv + 2uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} uv \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) + \\
& \frac{1}{6} uv \left( 5u^2 v^2 + 6 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right. \\
& \quad uv \left( 3uv + 2uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} uv \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) + \\
& \quad \left. \frac{1}{2} \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( 3u^2 v^2 + 4 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right. \right. \\
& \quad \left. \left. \frac{1}{3} uv \left( 3uv + 2uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} uv \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) \right) + \\
& \frac{1}{5} uv \left( 5uv + 4uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{3}{2} uv \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right. \\
& \quad \left. \frac{2}{3} \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( 3uv + 2uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} uv \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) + \\
& \quad \left. \frac{1}{4} uv \left( 3u^2 v^2 + 4 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) + \right. \right. \\
& \quad \left. \left. \frac{1}{3} uv \left( 3uv + 2uv \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) + \frac{1}{2} uv \left( u^2 v^2 + 2 \left( -\frac{u^2}{2} - \frac{v^2}{2} \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right)
\end{aligned}$$

In[66]:= **FullSimplify**[%]

Out[66]=  $\frac{1}{3628800}$   
 $( (-945 + 4725 u^2 - 3150 u^4 + 630 u^6 - 45 u^8 + u^{10}) (-945 + 4725 v^2 - 3150 v^4 + 630 v^6 - 45 v^8 + v^{10}) )$