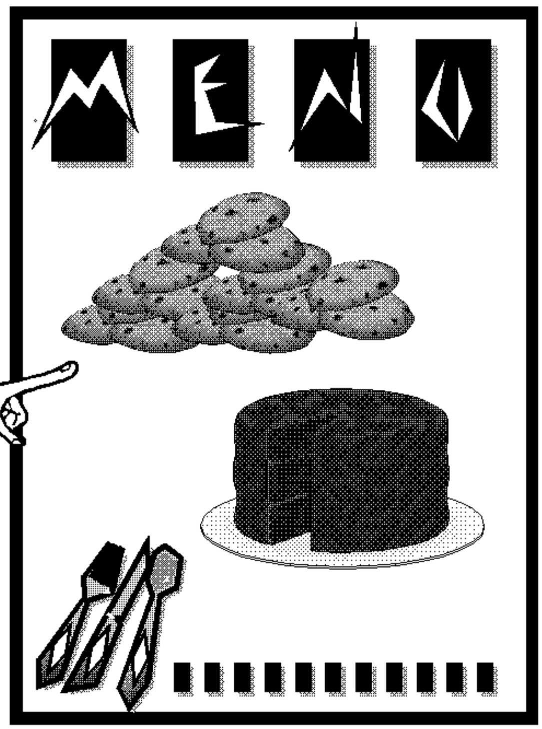
An Assortment of Longitudinal Data Analysis Examples and Problems



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MYTHS about longitudinal research

Rogosa, D. R. (1988). Myths about longitudinal research. In *Methodological issues in aging research*, K. W. Schaie, R. T. Campbell, W. M. Meredith, and S. C. Rawlings, Eds. New York, Springer Publishing Company, 171-209.

Rogosa, D. R. (1995). Myths and methods: "Myths about longitudinal research," plus supplemental questions. In The analysis of change, J. M. Gottman, Ed. Hillsdale, New Jersey: Lawrence Erlbaum Associates, 3-65.

1. Two Observations a longitudinal study make.
2. The difference score is intrinsically unreliable and unfair
3. You can determine from the correlation matrix for the longitudinal data whether or not you are measuring the same thing over time
4. The correlation between change and initial status is:
(a) *negative;* (b) *zero;* (c) *positive;* (d) *all of the above.*
5. You can't avoid regression toward the mean
6. Residual change cures what ails the difference score
7. Analyses of covariance matrices inform about change
7.1 Path analysis informs about change
7.2 Structural regression models inform about change
7.3 Simplex models describe most longitudinal data
8. Stability coefficients estimate:
(a) *the consistency over time of an individual;*
(b) *the consistency over time of an average individual;*
(c) *the consistency over time of individual differences;*
(d) *none of the above;* (e) *some of the above.*
9. Casual analyses support causal inferences about reciprocal effects

OLD BUSINESS

Conditional versus Unconditional Analyses (Goldstein, Plewis...)

[*UK Reading example*]

Longitudinal Data Examples

We have separate output sheets for the following examples:
Listings for Dental, Ramus, and (partial) Smearmiss given on "Data" sheet

Dental

From: lme and nlme: Mixed-effects Methods and Classes for S and S-plus Jose C. Pinheiro, Douglas M. Bates
Four measurements of the distance (in millimeters) from the center of the pituitary to the pteryomaxillary fissure made at ages 8, 10, 12, and 14 years on 16 boys and 11 girls (gender used as exogenous W).

Ramus

4 longitudinal observations on each of 20 cases. The measurement is the height of the mandibular ramus bone (in mm) for boys each measured at 8, 8.5, 9, 9.5 years of age.

WISC

4 observations, Wechsler Intelligence Scale for Children,Performance Scale, 86 children (times: begin first, end first, third, fifth grades). Gender is W

NC Fem

North Carolina Achievement Data (see Williamson, Applebaum, Epanchin, 1991). These education data are eight yearly observations on achievement test scores in math (Y), for 277 females each followed from grade 1 to grade 8, with a verbal ability background measure (W)

Smearmiss.

Artificial longitudinal data with known structure. Five observations (about 16% missing) on each of 100 individuals, with times of observation varying over individuals, and with an exogenous measure *W* for each individual.

Data Structures.

First four examples here have the simplest structure, no missing data, and "synchronous"--i.e., all individual measures at same times. In practice, data are missing; different observation times across individuals. Estimation procedures for the general case.

Models for Collections of Growth Curves

Straight-line Growth Curve Formulation.

attribute η, which exhibits systematic change over time. For individual p, growth curve in η is $\eta_p(t)$.

$$\eta_p(t) = \eta_p(0) + \theta_p t$$

Note: Rewrite using the centering parameter t^o ; θ and $\eta(t^o)$ are uncorrelated over the population of individuals $t^o = -\sigma_{\eta(0)\theta}/\sigma^2_\theta$

$$\eta_p(t) = \eta_p(t^o) + \theta_p(t - t^o) \quad .$$

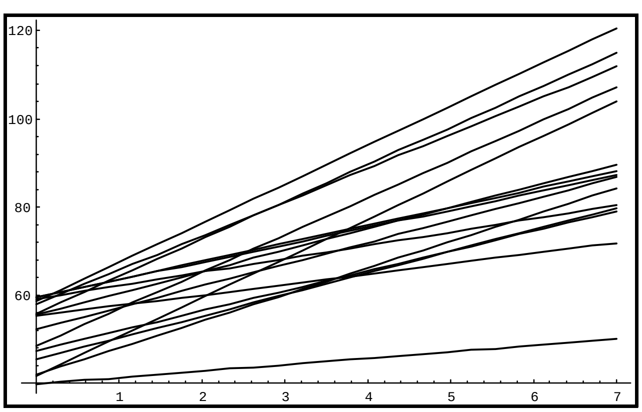
Constant rate of change θ_p -- first two moments $\mu_\theta \quad \sigma^2_\theta$
For systematic individual differences in growth (i.e. correlates of change) exogenous characteristic W. Conditional expectation
$E(\theta|W) = \mu_\theta + \gamma\,(W - \mu_W)$, With no measured exogenous variable, this between-unit model is $E(\theta|W) = \mu_\theta$.

Shown below 15 straight-line growth curves corresponding to pop. parameters $t^o = 2$; $\sigma^2_\theta = 5.333$; $\sigma^2_{\eta(t^o)} = 48$; $\theta \sim U[1, 9]$, $\eta(t^o) \sim U[38, 62]$. correlations among $\eta(t_i)$ for observation times $\rho_{\eta(1)\eta(4)} = .614$, $\rho_{\eta(1)\eta(6)} = .316$, $\rho_{\eta(4)\eta(6)} = .943$. For *Y*, var(ϵ) = 5, the pop. correlations are $\rho_{Y(1)Y(4)} = .567$, $\rho_{Y(1)Y(6)} = .297$, $\rho_{Y(4)Y(6)} = .894$.

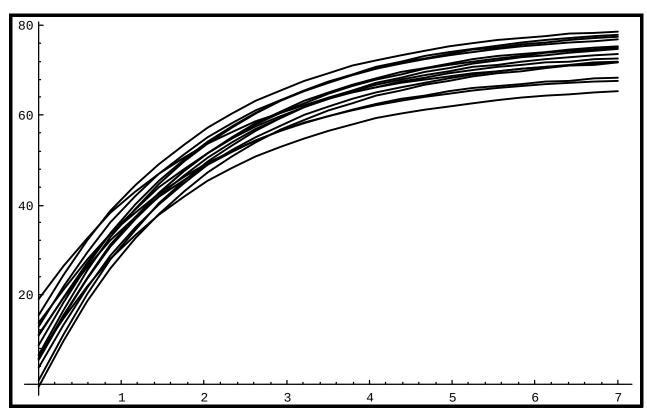
Alternative: exponential growth to an asymptote

Exponential growth curve with asymptote λ_p and curvature δ

Straight-line Growth



Exponential Growth



$$\eta_p(t) = \lambda_p - (\lambda_p - \eta_p(0))\exp(-\delta t) \quad .$$

exogenous variable *W* could be linked with both λ_p and $\eta_p(t^o)$.

Observables. oversimplified version-- observable Y is an imperfectly measured η, relation between Y and η is simple classical test theory model: $Y_p(t_i) = \eta_p(t_i) + \epsilon_i$

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longitudinal research questions and parameters of interest

1. *Individual and Group Growth.* Description of the form and amount of change, estimation of the individual (or group) growth curve, heterogeneity (individual differences) in the individual growth curves, and the statistical and psychometric properties of these estimates. **Parameters**: $f(\theta; t)$, μ_θ σ^2_θ $\rho(\hat{\theta})$ $\rho_{\eta(t_l)\theta}$

2. *Correlates and Predictors of Change.* systematic individual differences in growth e.g., "What kind of persons learn (grow) fastest?". **Parameters:** $\rho_{\theta W}$ $\beta_{\theta W}$

3. *Stability over Time.* consistency of individual differences over time. **Parameter:**

Foulkes-Davis γ = Pr(two growth curves do not intersect)

Common Claims:

*everyone changes at the same rate (people interchangable)

*change can't be measured reliably/accurately

*correlation of change and initial status is negative; regression toward mean pertains etc

Other research questions

4. Comparing Experimental Groups.

5. *Comparing Nonexperimental Groups.*

(note: Dental, Wisc compare intact groups via W code)

6. *Analysis of Reciprocal Effects.*

7. *Growth in Multiple Measures.* .

Data Analysis and Parameter Estimation

Precursor: Descriptive Growth Curve Analyses

SFYS: fit Y on t regressions, describe resulting $\hat{\theta}_p$, fit $\hat{\theta}_p$ on W regr,
Examples: WISC, frames 1-4; Ramus, frames 1-3; Dental, frames 1,2,4; SmearMiss, frames 1-3.
further, get variance components by approx method-of-moments (Rogosa-Saner 1995) works surprisingly well.

Maximum Likelihood estimation for parameters

General strategy: get elements of 2x2 est. covariance matrix of θ and $\eta(0)$ for full or incomplete data. Substitute for derived quantities. Also include W when exists with separate run (fixed effects).

Special, simple case; Complete, Synchronous Data.
ml estimation equations for full data in closed form:
example estimation of var(theta) σ^2_θ
MSR_p is the mean squared residual for the fit to individual p and $\hat{\sigma}^2$ is Ave(MSR_p). Then the estimate for σ^2_θ can be written,
$\hat{\sigma}^2_\theta = SS(\hat{\theta}_p)/\text{"n"} - \hat{\sigma}^2/SSt$,
reliability estimate for empirical rate is formed by $\hat{\rho}(\hat{\theta}) = \hat{\sigma}^2_\theta / SS(\hat{\theta}_p)/\text{"n"}$.

properties of mle: bias, precision
bias table: ML and REML coming up.

From Growth Curves to Mixed(Random)-Effects Models

$\eta_p(t) = \eta_p(0) + \theta_p t$ Data $Y_{pj} = \eta_p(t_{pj}) + \epsilon_{pj}$

ordered times $j = 1 \ldots T_p$ # obs for p (missing or not)

intercepts and slopes may differ across $p = 1, \ldots, N$

No W

$E_p(\theta_p) = \mu_\theta$

$E_p(\eta_p(0)) = \mu_{\eta(0)}$

Random

$\eta_p(0) - \mu_{\eta(0)}$

$\theta_p - \mu_\theta$

Fixed

$\mu_{\eta(0)}$

μ_θ

Mixed Effects Model

$$\underset{\sum_{p=1}^{N} T_p \times 1}{Y} = \underset{\sum T_p \times 2}{X}\,\underset{2\times 1}{\beta} + \underset{\sum T_p \times 2N}{Z}\,\underset{2N \times 1}{\gamma} + \epsilon$$

fixed

avc. growth curve

$$X = \begin{bmatrix} \begin{bmatrix} 1 & t_{p1} \\ \vdots & \\ 1 & t_{pT} \end{bmatrix} \\ \\ \end{bmatrix} \qquad \beta = \begin{bmatrix} \mu_{\eta(0)} \\ \mu_\theta \end{bmatrix}$$

random

$$\gamma = \begin{bmatrix} \begin{bmatrix} \eta_p(0) - \mu_{\eta(0)} \\ \theta_p - \mu_\theta \end{bmatrix} \\ \\ \end{bmatrix}$$

2 rows each p

blocks

$$Z = \begin{bmatrix} \begin{bmatrix} 1 & t_{p1} \\ \vdots & \\ 1 & t_{pT} \end{bmatrix} & & \\ & \ddots & \\ & & [\quad] \end{bmatrix}$$

$$Var(Y) = V = Z \underset{\text{also aka } D}{G} Z' + R$$

block elements of G

$$\begin{bmatrix} \sigma^2_{\eta(0)} & \sigma_{\eta(0)\theta} \\ & \sigma^2_\theta \end{bmatrix}$$

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With W

Fixed

$$E(\eta(0)|W) = \mu_{\eta(0)} + \beta_{\eta(0)W}(W - \mu_W)$$

$$E(\theta|W) = \mu_\theta + \beta_{\theta W}(W - \mu_W)$$

Random

$$\eta_p(0) - E(\eta(0)|W)$$

$$\theta_p - E(\theta|W)$$

$$Y = X\beta + Z\gamma + \varepsilon$$

fixed

expanding

$$\underset{\Sigma T_p \times 4}{X} = \begin{bmatrix} \begin{bmatrix} 1 & t_{p1} & w_p & w_p * t_{p1} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & t_{pT_p} & w_p & w_p * t_{pT_p} \end{bmatrix} \\ \\ \end{bmatrix}$$

$$\underset{4\times 1}{\beta} = \begin{array}{c} \begin{bmatrix} \mu_{\eta(0)} - \beta_{\eta(0)W}\,\mu_W \\ \mu_\theta - \beta_{\theta W}\,\mu_W \\ \beta_{\eta(0)W} \\ \beta_{\theta W} \end{bmatrix} \end{array} \begin{array}{l} \text{int} \\ \text{time} \\ W \\ \text{time}*W \end{array}$$

random

$$\underset{2N\times 1}{\gamma} = \begin{bmatrix} \begin{bmatrix} \eta_p(0) - E(\eta(0)|w_p) \\ \theta_p - E(\theta|w_p) \end{bmatrix} \\ \vdots \end{bmatrix}$$

G-matrix contains conditional variances

$Var(\theta|W) \quad Var(\eta(0)|W)$

see NCFem, Frames 7, 8 etc

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Implementation of Estimation using SAS- PROC MIXED

(thanks to Neil Timm, Univ Pitt. & Russ Wolfinger, SAS Inc)
REML default; ML available. (REML matches other E-M programs, e.g SmearMiss HLM).

S-plus Alternative: `lme`-- Pinheiro & Bates, or further with `nlme`
http://netlib.bell-labs.com/cm/ms/departments/sia/project/nlme/index.html

put data in column form [ID, Y, t, W]
Run PROC MIXED without and with W to obtain core quantities for parameter estimation

```
/* Proc mixed run */
 proc mixed data=yt;
   class case;
   model y = time / s;
   random int time / type=un sub=case gcorr;
   make 'CovParms' out=untot;
   make 'SolutionF' out=solfout;
   %bystmt;
 run;
```

From no-W run obtain Covariance Parameter Matrix (G);

```
 proc mixed data=yt;
   class case;
   model y = time W time*W / s;
   random int time / type=un sub=case gcorr;
   make 'SolutionF' out=solfout;
   %bystmt;
 run;
```

fixed effects solution vector gives relations with W

Raw SAS--- frames 7,8 NCFem; frame 6 Ramus; frames 7,8 Smearmiss;
frame 8 Dental.

TPSAS--

* obtain estimates for growth curve quantities of interest from solutions (using Make, ODS facility for 6.11) estimated covariance parameters give t^o , κ, variances and derived quantities etc; give relations on slide

* embed in jackboot.sas to obtain BCa confidence intervals for derived quantities

Extensions using properties of collections of growth curves

To estimate growth-curve quantities, substitute core estimates into these relations etc

variance

$$\sigma^2_{\eta(t)} = \sigma^2_{\eta(t^o)} + ((t - t^o)/\kappa)^2 \sigma^2_{\eta(t^o)}$$

covariance (also yields correlation, using above)

$$\sigma_{\eta(t_1)\eta(t_2)} = \sigma^2_{\eta(t^o)} + (t_1 - t^o)(t_2 - t^o)\sigma^2_{\theta}$$

correlation between change and status

$$\rho_{\eta(t)\theta} = \frac{(t - t^o)}{[\kappa^2 + (t - t^o)^2]^{1/2}}$$

correlation between exogenous variable, W and status

$$\rho_{W\eta(t)} = \frac{(t - t^o)\rho_{W\theta} + \kappa\rho_{W\eta(t^o)}}{[\kappa^2 + (t - t^o)^2]^{1/2}}$$

Time Path Output, in each data example, constructed from SAS (reml or ml) core estimates.

Bootstrap results array, in each data example, constructed by reformatting output from jackboot.sas (next page). Choose quantities to bootstrap... Examples: SmearMiss, frames 5-6 (ml and reml results); Ramus frame 5.

Standard Errors and Confidence Intervals via Bootstrap
(from TPSAS Rogosa/Ghandour/Kupermintz)
use PROC MIXED as core of %ANALYZE That jackboot calls

JACKBOOT.SAS
http://www.sas.com/techsup/download/stat/jackboot.sas
http://www.sas.com/service/techsup/faq/stat_macro/jackboot.html

name: jackboot
title: Jackknife and Bootstrap Analyses

Introduction

The %JACK macro does jackknife analyses for simple random samples, computing approximate standard errors, bias-corrected estimates, and confidence intervals assuming a normal sampling distribution.

The %BOOT macro does elementary nonparametric bootstrap analyses for simple random samples, computing approximate standard errors, bias-corrected estimates, and confidence intervals assuming a normal sampling distribution. Also, for regression models, the %BOOT macro can resample either observations or residuals.

The %BOOTCI macro computes several varieties of confidence intervals that are suitable for sampling distributions that are not normal.

If the %ANALYZE macro uses the %BYSTMT macro, two output data sets are created by the %JACK macro:
JACKDATA contains the jackknife resamples. The variable _SAMPLE_ gives the resample number, and _OBS_ gives the original observation number.
JACKDIST contains the resampling distributions of the statistics in the OUT= data set created by the %ANALYZE macro. The variable _SAMPLE_ gives the resample number.

Two similar data sets are also created by the %BOOT macro when the %BYSTMT macro is used:
BOOTDATA contains the bootstrap resamples. The variable _SAMPLE_ gives the resample number, and _OBS_ gives the original observation number.
BOOTDIST contains the resampling distributions of the statistics in the OUT= data set created by the %ANALYZE macro. The variable _SAMPLE_ gives the resample number.

mle and reml simulation (50,000); complete synchronous data

Estimation of var(theta)

true	n	ML		REML	
5.0	10	4.37	[7.39]	4.99	[8.61]
	15	4.58	[5.06]	4.99	[5.59]
	25	4.76		5.01	

true	n	ML	REML
3.0	10	2.59	3.14
	15	2.67	3.06
	25	2.78	3.01

Estimation of Rel(theta-hat)

true	n	ML	REML
.806	10	.726	.752
	15	.758	.774
	25	.781	.789

true	n	ML	REML
.50	10	.379	.425
	15	.407	.440
	25	.439	.460

ASSESSMENTS OF STABILITY.

Questions about temporal stability fall into two broad headings--
Is an individual consistent over time? and *Are individual differences consistent over time?* (Rogosa, Willett, and Floden 1984) .

index of tracking γ proposed by Foulkes and Davis (1981):
assess consistency of individual differences over a specified time interval.
the index estimates the probability that two randomly chosen individual's trajectories do not cross in the time interval specified.
"tracking" if index > .50 (significantly).

Estimation. Fit individual trajectories (straight-line or polynomial etc).
For each individual compute the proportion of other trajectories not crossed.
Point estimate is the average over individuals of these proportions.
F-D p.441 use standard deviation of individual estimates divided by Sqrt[n] as the standard error and construct normal theory CI.

odd?? Bootstrap s.e. typically is almost exactly twice as large as F-D.

RAMUS DATA

```
    est.   mean.boot   se.boot   s.e. F-D
 0.8263    0.7858    0.0655       .0320
$CI:         0.025      0.05      0.95     0.975
  Standard 0.6978    0.7185    0.9341    0.9547
Percentile 0.6421    0.6684    0.8842    0.8947
       BCa 0.7210    0.7473    0.9158    0.9263
```

WISC DATA

```
    est.   mean.boot   se.boot   s.e. F-D
 0.6700    0.6601    0.0367       .0185
```

NC Fem DATA

```
    est.   mean.boot   se.boot   s.e. F-D
 0.7212    0.7185    0.0174       .0087
```

RAT DATA (n=10)

```
    est. mean.boot   se.boot   s.e. F-D
 0.6222    0.5248    0.100       .0529
```

ARTIFICIAL DATA (n=200)

```
    est.  mean.boot  se.boot   s.e. F-D
 0.6930    0.679     0.0445      .0223
```

What About Time-1, Time-2 Data?

*Fitting straight-line to 2 (noisy) data points

*sample quantitity Correlation(Y1, Y2 - Y1) badly biased estimate for correlation change and initial status.
Examples:
Myths Table 1.6. Artificial Data set-up; true =.55, E(sample) = -.12
Corresponding Artificial data sample (full data) gives .06

Dental: sample 1,4 = -.33 ; est true is .209
NCFem: Grades 1 to 8 change: sample is .07;
estimated true is .65 (frame 4)

Notes on Data Analysis Examples

small n, estimation imprecise (most often).
important to report s.e., CI (e.g. HLM crowd n=10)
examples: WISC (n=86) frames 6-7, Corr(Rate, Initial Status), var(Rate)
Dental (n=27) rel(Rate) in {0,.8},
Corr(Rate, Initial Status) in {-.4,1.0}
Ramus (n=20) does better than Dental

Corr(Rate, Initial Status) can be positive and large
examples NCFem, WISC?

Standard error of exogenous variable regression
OLS = Reml? see NCFem frames 6-8

Change can be assessed accurately, reliably

Other Lessons

Assorted References

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